

VDH · VALUATION DISCUSSION HOUR

Time Value of Money

Exercise Sheet & Memorandum

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16 exam-style questions with fully worked solutions | Total: 102 marks

Companion to the VDH TVM Calculator and Formula Sheet.

Part 1 — Exercise Sheet

Work through the questions below. Show your formulae and intermediate factors. Answers may differ slightly depending on rounding — carry factors to at least four decimal places. All rates are per annum unless stated. Marks are shown in brackets.

Notation. PV = present value · FV = future value · i = periodic rate · n = number of periods · A = periodic payment · YP = Years' Purchase (the annuity/perpetuity multiplier). Refer to the VDH **Formula Sheet** for the full set of formulae.

GROUP A — SINGLE SUMS (COMPOUNDING & DISCOUNTING)

Question 1. Future value of a single sum.

[4]

An investor deposits **R250,000** into an account earning **8% per annum, compounded annually**. What will the deposit be worth after **6 years**?

Question 2. Discounting a future sale price.

[4]

A property is expected to sell for **R4,500,000** in exactly **5 years**. Using a discount rate of **11% per annum**, calculate the present value of that future receipt.

Question 3. Solving for the rate of return.

[5]

An investment of **R1,200,000** grew to **R1,800,000** over **7 years** with annual compounding. What compound annual rate of return was achieved?

Question 4. Solving for the holding period.

[5]

How long, to the nearest year, will it take for **R500,000** to **double** in value at **9% per annum**, compounded annually?

GROUP B — ANNUITIES & YEARS' PURCHASE

Question 5. Present value of an income stream (Years' Purchase).

[6]

A property produces a net rental income of **R360,000 per annum, receivable annually in arrears**, for a fixed term of **10 years**. Using a rate of **12% per annum**, calculate the present value of the income stream and state the Years' Purchase (YP) factor used.

Question 6. Annuity due (rent in advance).

[4]

Reconsider Question 5, but assume the rent of **R360,000** is receivable **annually in advance**. What is the present value now?

Question 7. Sinking fund contribution.

[6]

A body corporate must accumulate **R2,000,000** to replace a roof in **8 years' time**. Equal contributions are invested at the **end of each year** in a fund earning **7% per annum**. What annual contribution is required?

GROUP C — LOAN AMORTISATION

Question 8. Monthly bond instalment.

[6]

A **R3,000,000** mortgage bond is granted at **10.5% per annum, compounded monthly**, repayable over **20 years**. Calculate the monthly instalment.

[7]

Question 9. Outstanding balance.

For the bond in Question 8, calculate the outstanding balance immediately after the **60th** monthly payment (i.e. after 5 years).

GROUP D — DISCOUNTED CASH FLOW & IRR**Question 10.** Net present value of a property acquisition.**[12]**

An investor is offered a property for **R5,000,000** (payable now, Year 0). It is forecast to produce net cash flows at the end of Years 1–5, and to be sold for **R6,000,000** at the end of Year 5. Using a target rate of **13% per annum**, calculate the NPV and advise whether the purchase meets the target.

Year	1	2	3	4	5
Net cash flow (R)	480,000	520,000	560,000	600,000	640,000

(The Year 5 sale proceeds of R6,000,000 are in addition to the R640,000 income.)

Question 11. Internal rate of return by interpolation.**[8]**

Using the cash flows in Question 10, estimate the IRR by **linear interpolation**, given that the NPV at 13% is **+R192,040** and the NPV at **18%** is **–R667,053**. Comment briefly on the accuracy of the estimate.

GROUP E — CAPITALISATION & PERPETUITIES**Question 12.** Capitalisation in perpetuity.**[5]**

A fully-let, stabilised property produces a net income of **R840,000 per annum**, assumed to continue in perpetuity. Using an all-risks yield of **9%**, calculate the capital value and state the Years' Purchase in perpetuity.

Question 13. Growing perpetuity.**[5]**

Assume instead the net income of **R840,000** (receivable in one year) is expected to grow at **3% per annum** in perpetuity. Using a discount rate of **11%**, calculate the capital value.

Question 14. Deferred perpetuity (reversion).**[6]**

The right to receive a perpetual net income of **R600,000 per annum** commences in the future, with the **first receipt at the end of Year 5**. Using a rate of **10%**, calculate the present value today.

Question 15. Effective annual rate.**[4]**

A bank quotes a nominal rate of **10.5% per annum, compounded monthly**. Calculate the effective annual rate (EAR).

GROUP F — CAPSTONE**Question 16.** Term-and-reversion freehold valuation.**[15]**

A commercial building is let on a lease with **4 years unexpired** at a passing (contract) rent of **R1,200,000 per annum, in arrears**. The current market rental value (MRV) is **R1,500,000 per annum**. On expiry the property re-lets at MRV and is assumed to produce that income **in perpetuity**. Value the freehold interest by the term-and-reversion method:

- capitalise the **term** income at **9%**;
- capitalise the **reversion** to perpetuity at **9.5%**, deferred 4 years at 9.5%.

Part 2 — Memorandum (Worked Solutions)

Full solutions follow. Intermediate factors are shown to six decimal places and final money figures to the nearest cent; sensible rounding of reported values is normal practice.

Solution 1 — Future value of a single sum

$$FV = PV (1 + i)^n = 250,000 \times 1.08^6$$

$$1.08^6 = 1.586874 \rightarrow FV = 250,000 \times 1.586874 \quad \mathbf{FV = R396,718.58}$$

Solution 2 — Present value of a future receipt

$$PV = FV \div (1 + i)^n = 4,500,000 \div 1.11^5$$

$$1.11^5 = 1.685058 \rightarrow PV = 4,500,000 \div 1.685058 \quad \mathbf{PV = R2,670,530.98}$$

Solution 3 — Compound annual rate

$$i = (FV \div PV)^{1/n} - 1 = (1,800,000 \div 1,200,000)^{1/7} - 1$$

$$= 1.5^{0.142857} - 1 = 1.059634 - 1 \quad \mathbf{i = 5.96\% \text{ per annum}}$$

Solution 4 — Holding period to double

$$n = \ln(2) \div \ln(1 + i) = 0.693147 \div \ln(1.09) = 0.693147 \div 0.086178$$

$n = \mathbf{8.04 \text{ years}}$. After 8 full years the balance is $500,000 \times 1.09^8 = R996,281$ (just short of double); after 9 years it is R1,085,947.

$\approx \mathbf{8.04 \text{ years} — i.e. 9 \text{ full years to be certain of doubling}}$

Solution 5 — PV of a 10-year income stream (YP)

$$YP = [1 - (1 + i)^{-n}] \div i = [1 - 1.12^{-10}] \div 0.12$$

$$= [1 - 0.321973] \div 0.12 = 0.678027 \div 0.12 = \mathbf{5.65022}$$

$$PV = 360,000 \times 5.65022 \quad \mathbf{PV = R2,034,080.29 \text{ (YP = 5.65022)}}$$

Solution 6 — Annuity due (rent in advance)

$$PV_{\text{due}} = PV_{\text{ordinary}} \times (1 + i) = 2,034,080.29 \times 1.12$$

$$\text{Equivalently, YP in advance} = 5.65022 \times 1.12 = 6.328247. \quad \mathbf{PV = R2,278,169.93}$$

Solution 7 — Sinking-fund contribution

$$A = FV \times i \div [(1 + i)^n - 1] = 2,000,000 \times 0.07 \div (1.07^8 - 1)$$

$$1.07^8 = 1.718186 \rightarrow A = 140,000 \div 0.718186 \quad \mathbf{A = R194,935.52 \text{ per annum}}$$

Solution 8 — Monthly bond instalment

$i = 0.105 \div 12 = 0.00875$ per month; $n = 20 \times 12 = 240$.

$$A = P \times i \div [1 - (1 + i)^{-n}] = 3,000,000 \times 0.00875 \div [1 - 1.00875^{-240}]$$

$$1.00875^{240} = 8.091918 \rightarrow \text{denominator} = 1 - 0.123580 = 0.876420$$

$$A = 26,250 \div 0.876420 \quad \mathbf{\text{Monthly instalment} = R29,951.40}$$

Solution 9 — Outstanding balance after 60 payments

Remaining term $n - k = 240 - 60 = 180$ payments.

$$\text{Balance} = A \times [1 - (1 + i)^{-(n-k)}] \div i = 29,951.40 \times [1 - 1.00875^{-180}] \div 0.00875$$

$$1.00875^{180} = 4.797761 \rightarrow \text{factor} = 0.791567 \div 0.00875 = 90.46508$$

$$\text{Balance} = 29,951.40 \times 90.46508 \quad \mathbf{\text{Outstanding balance} = R2,709,555.43}$$

Cross-check (retrospective): $3,000,000 \times 1.00875^{60} - 29,951.40 \times [(1.00875^{60} - 1) \div 0.00875] = R2,709,555.43 \checkmark$

Solution 10 — NPV of the acquisition

Year	Cash flow (R)	PV factor @13%	Present value (R)
1	480,000	0.884956	424,778.76
2	520,000	0.783147	407,236.28
3	560,000	0.693050	388,108.09
4	600,000	0.613319	367,991.24
5	6,640,000	0.542760	3,603,925.98
PV of inflows			5,192,040.34

$$\text{NPV} = -5,000,000 + 5,192,040.34 \quad \mathbf{\text{NPV} = +R192,040.34}$$

The NPV is positive, so the investment yields more than the 13% target — the purchase is justified at that rate.

Solution 11 — IRR by linear interpolation

$$\text{IRR} \approx r_1 + \text{NPV}_1 \div (\text{NPV}_1 - \text{NPV}_2) \times (r_2 - r_1)$$

$$= 13\% + 192,040 \div [192,040 - (-667,053)] \times (18\% - 13\%)$$

$$= 13\% + 192,040 \div 859,093 \times 5\% = 13\% + 0.2235 \times 5\% = 13\% + 1.12\%$$

$$\mathbf{\text{IRR} \approx 14.12\%}$$

Interpolation slightly overstates the true IRR ($\approx 14.02\%$) because NPV is a convex function of the discount rate; using a narrower rate bracket around the root reduces the error.

Solution 12 — Capitalisation in perpetuity

Value = Income $\div$ i = 840,000 $\div$ 0.09 ; YP in perpetuity = 1 $\div$ i = 1 $\div$ 0.09

Value = R9,333,333.33 (YP = 11.1111)

Solution 13 — Growing perpetuity

Value = $I_1 \div (r - g)$ = 840,000 $\div$ (0.11 - 0.03) = 840,000 $\div$ 0.08

Value = R10,500,000.00

Solution 14 — Deferred perpetuity (reversion)

A perpetuity valued at time t capitalises payments beginning at $t+1$. First receipt is at end of Year 5, so value at end of Year 4:

Value_{yr4} = 600,000 $\div$ 0.10 = R6,000,000 ; PV = 6,000,000 $\div$ 1.10⁴

1.10⁴ = 1.464100 $\rightarrow$ PV = 6,000,000 $\div$ 1.464100 **PV = R4,098,080.73**

Solution 15 — Effective annual rate

EAR = $(1 + j \div m)^m - 1$ = $(1 + 0.105 \div 12)^{12} - 1$ = 1.00875¹² - 1

= 1.110203 - 1 **EAR = 11.02% per annum**

Solution 16 — Term-and-reversion freehold valuation

Term — R1,200,000 for 4 years @ 9%:

YP 4 yrs @ 9% = $[1 - 1.09^{-4}] \div 0.09$ = $[1 - 0.708425] \div 0.09$ = 3.23972

Term value = 1,200,000 $\times$ 3.23972 = **R3,887,663.85**

Reversion — R1,500,000 in perpetuity @ 9.5%, deferred 4 years:

Perpetuity value = 1,500,000 $\div$ 0.095 = R15,789,473.68

Deferred 4 yrs: $\times 1.095^{-4}$ = $\times 0.695574$

Reversion value = 15,789,473.68 $\times$ 0.695574 = **R10,982,752.00**

Component	Value (R)
Term (4 yrs @ 9%)	3,887,663.85
Reversion (perpetuity @ 9.5%, deferred 4 yrs)	10,982,752.00
Freehold value	14,870,415.85

Freehold value $\approx$ R14,870,000 (say R14.87m) Note: a two-rate term-and-reversion

approach is used — a lower yield on the more secure term income and a slightly higher yield on the reversion. Rounding of the final figure to an appropriate degree of precision is normal valuation practice.

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